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Engineering Notes

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Computation of a Science Orbit About Europa

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Introduction

THE instrument requirements for a scientific mission about Europa constrain the orbit design to a subset of limited values of the orbital elements. Near-circular, low-altitude, high-inclination orbits are normally required for mapping missions, and space-mission designers try to minimize the altitude variation of the satellite over the surface of the body by searching for orbits with small eccentricity and with a fixed argument of periapsis. These orbits are usually called frozen orbits [1–3].

However, due to third-body perturbations, high-inclination orbits around planetary satellites are known to be unstable [4–6], and thus emerges the problem of maximizing the orbital lifetime. One proposed approach to maximize orbit lifetime resorts to dynamical systems theory. This approach has been shown to be useful in orbit maintenance routines, in which the stable manifold associated with unstable nominal orbits provides a efficient way of maximizing time between maneuvers [7]. In the same fashion, paths in the plane of argument of periapsis and eccentricity that yield long lifetime near polar orbits around Europa have been recently identified [8].

In this paper we take a different approach. Periodic orbits around Europa are known to exist and have been previously used in the investigation of stability regions around Europa [9–11]. Periodic orbits in the rotating frame are ideal, nominal, repeat ground-track orbits that, for long enough repetition cycles, are suitable for mapping missions. We compute low-altitude, near-circular, highly inclined, repeat ground-track, unstable periodic orbits, and find that these kinds of solutions enjoy long lifetimes. Our procedure is based on fast numerical algorithms that are easily automated. The numerical search for initial conditions of repeat ground-track orbits is very simple and feasible even for higher-order gravity fields [12,13]. Safe recurrences for computing the gradient and Hessian of the gravitational potential can be found in the reference list (see, for instance, [14] and references therein).

For our search, we use a simplified dynamical model that considers the mean gravitational field of a synchronously rotating and orbiting moon, and take into account the perturbations of the third body in the Hill problem approximation [5,15,16]. Tests on the validity of the solutions are made in an ephemeris model that includes perturbations of the sun, the other Galileans, the nonsphericity of Jupiter, the other gas giants, and a Europa gravity model that is consistent with synchronous moon theory and NASA's Galileo close encounters [17].

In passing from the simplified to ephemeris model we introduce a one-dimensional parameter scaling of the initial conditions that proved efficient in the past [10]. On one side it provides a simple and feasible optimization for a given epoch. But it also shows how isolated the optimized solution is in the ephemeris model, thus giving a reasonable estimation of the robustness of the solution in the presence of realistic perturbing forces.

With respect to the Europa gravity field, it turns out that the Galileo flyby data cannot detect valid signatures for any gravitational terms for Europa beyond μ, J_2, and $C_{2,2}$ [17]. However, based on observations of other celestial bodies, it seems reasonable to speculate that Europa could be top or bottom heavy [18], and previous studies have shown that J_3 can play an important role [8,18,19]. Therefore, we study the influence of J_3 in the proposed orbits, and find repeat ground-track orbits with higher eccentricities than the second-order gravity field solutions, but with similar lifetimes.

Dynamical Model

The restricted three-body problem retains the underlying dynamics that govern the motion of an orbiter about planetary satellites. Close to the planetary satellite its physical nature in terms of a gravitational harmonics expansion must be considered. Further, the Hill problem can be used to model the planetary perturbation on a satellite orbiter when the planet-satellite distance is sufficiently large. Then, in the rotating frame of reference with origin at the center of mass of the satellite, using a Hamiltonian formulation we write

$$\mathcal{H} = (1/2)(\boldsymbol{X}\cdot\boldsymbol{X}) - \boldsymbol{\omega}\cdot(\boldsymbol{x}\times\boldsymbol{X}) - (\mu/r) - R(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x} = (x, y, z)$ is the position vector of the orbiter, $r = \|\boldsymbol{x}\|$, $\boldsymbol{X} = (X, Y, Z)$ is the vector of conjugate momenta (the velocity in the inertial frame), $\boldsymbol{\omega}$ is the angular velocity of the satellite around the planet, μ is the satellite's gravitational parameter, and the perturbing function R includes the third body and nonsphericity perturbations.

For synchronous orbiting and rotating planetary satellites, the tidal force due to the planet is static in a body-fixed reference frame. Over time, this direct force imparts a nonspherical shape on the satellite, and to first order, equilibrium theory predicts this shape becomes a triaxial ellipsoid with the longest axis aligned towards the planet. This shape is well approximated with the J_2 and $C_{2,2}$ terms of the standard spherical harmonic expansion of the potential. Then, the perturbing function is

$$R = (\omega^2/2)(3x^2 - r^2) + (\mu/r)N(\boldsymbol{x}) \tag{2}$$

For the nondimensional part N of the perturbing potential of the satellite, we consider

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$$N = \frac{\alpha^2}{r^2}\left[J_2\left(\frac{1}{2} - \frac{3z^2}{2r^2}\right) + 3C_{2,2}\frac{x^2 - y^2}{r^2}\right] + J_3\frac{\alpha^3}{r^3}\frac{z}{r}\left(\frac{3}{2} - \frac{5z^2}{2r^2}\right) \quad (3)$$

where α is the equatorial radius of the satellite, and J_2, $C_{2,2}$, and J_3 are the harmonic coefficients with physical meaning representing oblateness, ellipticity, and latitudinal asymmetry, respectively. As we assume equilibrium theory, we set $C_{2,2} = (3/10)J_2$ [20]. Despite the fact that J_3 is null in equilibrium theory, similar to other celestial bodies Europa could be top or bottom heavy. Therefore, we retain J_3 in the simplified model to investigate the modifications that it could introduce in specific solutions.

Averaging of Eq. (1) for different perturbing functions has proven very useful in studying the long term behavior around Europa [5,6,8,21]. The main dynamics close to planetary satellites correspond to the Hill problem where circular orbits suffer from instability for inclinations that depart the equatorial plane by more than $\sim$40 deg [22]. Perturbations due to the nonsphericity of the central body only introduce minor qualitative changes in the long term dynamics [6,9,15,16] except that they can significantly affect the orbit eccentricity. However, the candidates for a Europa science orbit are among the unstable orbits. Thus, it is essential to consider the short period dynamics in the search for long lifetime orbits [8]. It seems natural, then, to proceed directly with the nonaveraged problem Eq. (1), where periodic orbits are the particular solutions that will reach the longest lifetimes. This is the approach we take here. Periodic orbits around Europa indeed exist in abundance, and have been used previously for a variety of dynamical applications [9–11]. Thus, we search for periodic orbits of Eq. (1) that satisfy the requisites of a science orbit about Europa. We used the methodology described in [9,10] and refer the interested reader therein.

Families of Periodic Orbits

We assume inclination limits $70 < i < 110$ deg for the scientific mission, and assume also that it requires an altitude of approximately 100 km over the surface of Europa. Therefore, we limit ourselves to computing the families of orbits that close after 41 and 40 cycles in one Europa day, and 81 cycles in two Europa days (1:42-, 1:41-, and, 2:83-resonant families, respectively, in the terminology of [10]), which, as Fig. 1 shows, can satisfy these requirements. Higher-order resonances will provide many other orbits satisfying those requisites. The three families are made of low-eccentricity orbits with unstable character in the range of inclinations $49 < i < 133.5$ deg. In what follows, we only present results for orbits of the 1:41-resonant family. Experiments performed with orbits of the other families produced very similar results.

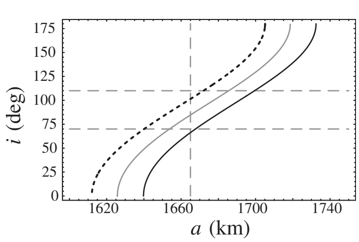

Fig. 1 1:42- (dotted), 2:83- (gray), and 1:41- (black) resonant families. Averaged elements.

Figure 2 shows a sample orbit of the 1:41-resonant family with averaged orbital elements $a = 1683.217$ km, $e = 0.0009$, $i = 90.001$ deg. The evolution of the instantaneous orbital elements in one period is also presented, where two kinds of short period effects are observed. One produces fast oscillations of the orbital elements with half the period between two consecutive upward crossings of the equatorial plane of the periodic orbit; these oscillations scarcely affect the instantaneous inclination. These small amplitude, very short period oscillations are modulated by an oscillation with half the frequency of the periodic orbit, the smaller amplitudes corresponding to the times where the orbital plane is perpendicular to the x–z plane.

Note that the periodic orbits are periodic only within certain numerical precision $\epsilon = \max|\xi_i(T) - \xi_i(0)|$ $(i = 1, \ldots, 6)$, where ξ_i stands for any of the coordinates in phase space. The periodicity error ϵ together with rounding errors play the role of small perturbations in a long term propagation, where the orbiter enters the unstable manifold of the unstable periodic orbit. Then, the exponential increase of the eccentricity forces the orbiter to impact Europa. Figure 3 shows two sample propagations for the polar orbit. In the left plot the orbit remains periodic for almost 1 year and delays the impact to Europa to more than 400 days. We started from a periodic orbit with periodicity $\epsilon < 10^{-11}$ in internal units, that amounts to better than 1 mm in position and 10^{-4} mm/s in velocity, which is highly unrealistic from a practical point of view. The periodicity of the starting orbit used in the right plot is of the order of 1 km in position and 1 m/s in velocity, and the lifetime diminishes to about 110 days.

To investigate how the possible nonuniform density and shape of Europa affects the behavior of the Europa science orbit, now we consider a pear-shaped Europa. We compute families of periodic orbits for variations of J_3 starting from the preceding solutions, and assuming a limit value $|J_3| = C_{2,2}$. From our computations we conjecture that J_3 does not worsen the stability behavior. This is expected because the primary origin of the dynamical instability of high-inclination orbits is the third-body perturbation on the central body attraction.

As J_3 breaks the equatorial symmetry, low-eccentricity periodic orbits no longer exist [2]. In agreement to [8], we note that negative

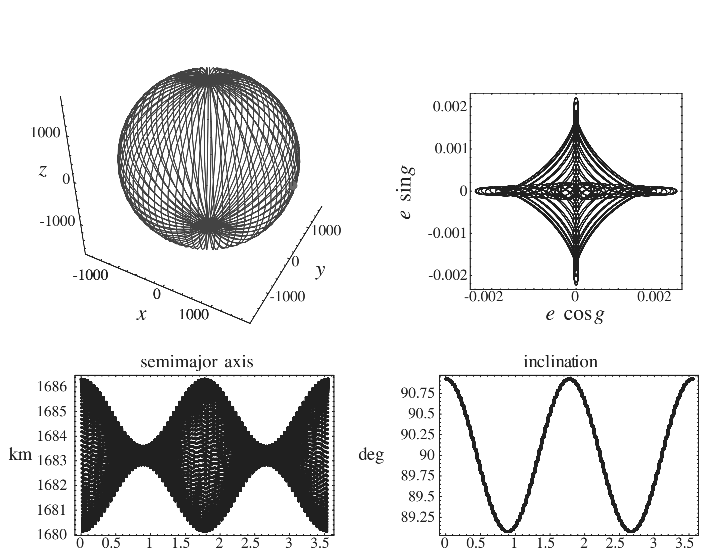

Fig. 2 Sample repeat ground-track orbit and evolution of the orbital elements.

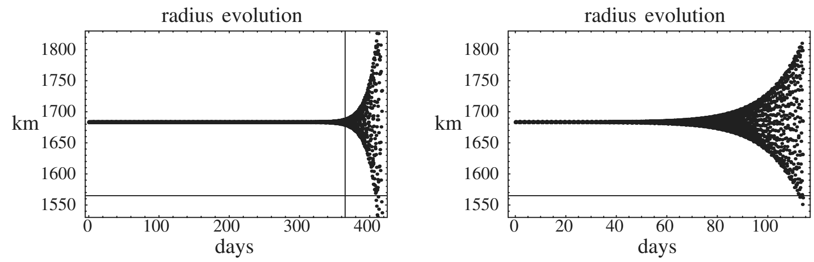

Fig. 3 Long term propagation. Case $J_3 = 0$. Left: $\epsilon < 10^{-11}$. Right: $\epsilon < 10^{-3}$.

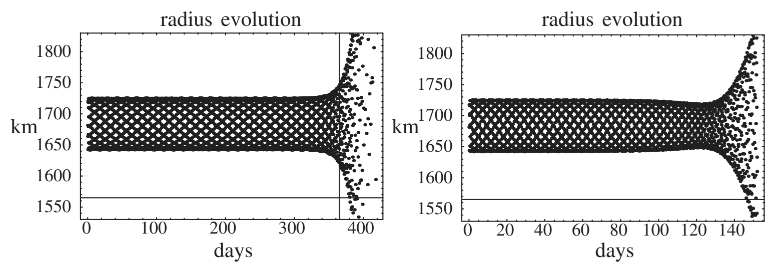

Fig. 4 Long term propagation. Case $J_3 \sim C_{2,2}$. Left: $\epsilon < 10^{-11}$. Right: $\epsilon < 10^{-3}$.

J_3 values will produce low-eccentricity orbits with $g = 90$ deg, whereas positive J_3 values produce low-eccentricity orbits with $g = 270$ deg. Except for having a higher value of the eccentricity, the instantaneous orbital elements behave very similar to the $J_3 = 0$ case, with the higher oscillations corresponding to instantaneous retrograde inclinations. Figure 4 shows similar examples to the case $J_3 = 0$, where similar lifetimes are reached.

Ephemeris Model

The ephemeris model is based on three publicly available estimated solutions for the parameters, positions, velocities, and orientations of celestial bodies of interest.[‡] The active bodies include the sun, the gas giant planets, and all four Galilean moons. A zonal Jovian gravity field is considered using J_2, J_3, J_4, and J_6 from the JUP230 ephemeris.[§] The only nonspherical Europa gravity terms are $J_2 = 4.355 \times 10^{-4}$ from [23] and $C_{2,2} = (3/10)J_2$ from synchronous moon theory.

For a given epoch, the inertial directions of the instantaneous Jupiter–Europa vector $\boldsymbol{\rho} = \boldsymbol{r}_E - \boldsymbol{r}_J$ and the system angular momentum $\boldsymbol{h} = \boldsymbol{\rho} \times (\boldsymbol{v}_E - \boldsymbol{v}_J)$ are used to define a coordinate rotation to the ephemeris system. Vectors $\boldsymbol{r}$ and $\boldsymbol{v}$ are position and velocity in the ephemeris system centered at the solar system barycenter, whereas subscripts E and J stand for Europa and Jupiter, respectively. Thus, the mapping $\mathcal{F}:(\boldsymbol{x}, \dot{\boldsymbol{x}}) \to (\boldsymbol{r}, \boldsymbol{v})$ from the rotating, Europa-centered frame to the inertial ephemeris frame is

$$\boldsymbol{r} = \boldsymbol{r}_E + \boldsymbol{R}\boldsymbol{x}, \qquad \boldsymbol{v} = \boldsymbol{v}_E + \dot{\boldsymbol{R}}\boldsymbol{x} + \boldsymbol{R}\dot{\boldsymbol{x}} \tag{4}$$

where the elements of the rotation matrix $\boldsymbol{R} = (\boldsymbol{u}_x, \boldsymbol{u}_y, \boldsymbol{u}_z)$ are

$$\boldsymbol{u}_x = \boldsymbol{\rho}/\|\boldsymbol{\rho}\|, \qquad \boldsymbol{u}_z = \boldsymbol{h}/\|\boldsymbol{h}\|, \qquad \boldsymbol{u}_y = \boldsymbol{u}_z \times \boldsymbol{u}_x$$

Long term propagations of the simplified initial conditions mapped into the ephemeris model provide the lifetimes of the selected orbits. Note, however, that the ephemeris runs are attached to a given epoch, and so are the conclusions regarding lifetimes. To test the general validity of the lifetimes we could make a variety of runs for different epochs. We prefer to proceed distinctly: we perform all the ephemeris runs for a fixed epoch (1 January 2025) and use a one-dimensional scaling parameter k that proved very useful in the past [10].

The k-scaling in the range $0.97 < k < 1.03$ accounts for variations in the distance from Europa to Jupiter, on the order of a few percent of its nominal value, produced by the nonzero eccentricity of the orbit of Europa and slight perturbations in semimajor axis. Before the transformation from Eq. (4), we scale the initial state to

$$\boldsymbol{x}' = k\boldsymbol{x}, \qquad \dot{\boldsymbol{x}}' = \dot{\boldsymbol{x}}\sqrt{1/k} \tag{5}$$

The nonlinear scaling of the velocity results from assuming Europa maintains its circular velocity although the Jupiter–Europa distance is slightly scaled. Consequently, assuming it remains periodic, the period of the scaled spacecraft orbit should be $T' = T\sqrt{k^3}$. After each scaling for a given k we perform the propagation and plot the resulting lifetime τ.

Figure 5 presents results corresponding the 40 cycle periodic orbit for different inclinations. Lifetimes τ average to about four months with peaks of more than six months, where the best results side with the polar orbits. The plot at the top corresponds to the k-scaling of initial conditions $(x, 0, 0, 0, \dot{y}, \dot{z})$, whereas at the bottom the initial conditions are close to the y-axis.

The case $k = 1$ typically results in repeat ground-track orbits of the ephemeris model, where the ground trace does not drift substantially from its nominal value. However, the lifetime optimization provided by the k-scaling normally destroys the repeat ground trace condition (see Fig. 6). This lack of repeating ground tracks may in fact be preferred from a mapping perspective because it provides a more complete global surface coverage. Similar results are obtained for the $J_3 \sim C_{2,2}$ case, where the lifetimes are not significantly affected.

Note that the curve $\tau = \tau(k)$ provides a reasonable estimation of the robustness of the solution, i.e., how sensitive is it to initial conditions in the ephemeris model. Further, the one-dimensional k-scaling can be seen as a feasible optimization of the initial state: that allowed us to find orbits with lifetimes of six months in the ephemeris model.

Because the ephemeris model contains many perturbations that are unaccounted for in our simple model, we find it difficult to predict what values of k will lead to the longest lifetime orbits. However, by picking an epoch and performing an educated one-dimensional search, we present a simple method for demonstrating and finding long lifetime ephemeris solutions with the underlying assumption that the ephemeris force model is known.

‡Data available on-line at http://naif.jpl.nasa.gov/naif/spiceconcept.html, ftp://naif.jpl.nasa.gov/pub/naif/generic_kernels/spk/satellites/jup230.bsp, ftp://naif.jpl.nasa.gov/pub/naif/generic_kernels/spk/planets/de405_2000-2050.bsp, and ftp://naif.jpl.nasa.gov/pub/naif/generic_kernels/pck/pck00008.tpc [cited 28 July 2006].

§Data available on-line at ftp://naif.jpl.nasa.gov/pub/naif/generic_kernels/spk/satellites/jup230.bsp [cited 28 July 2006].

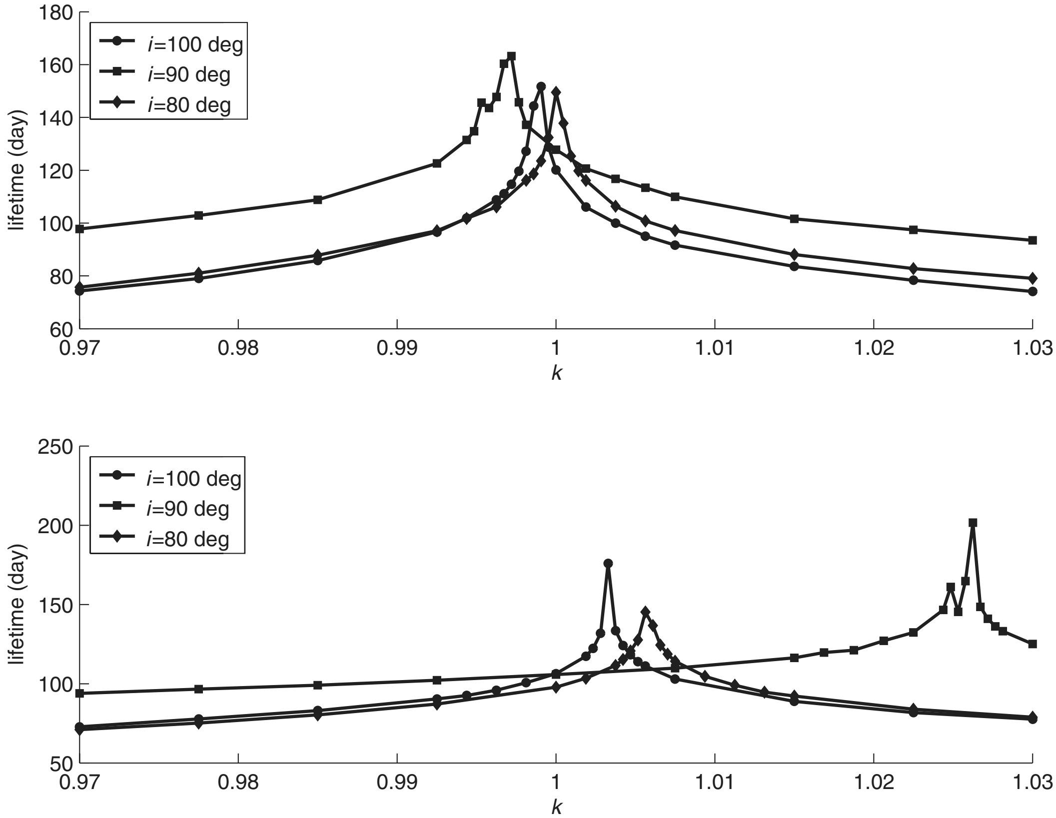

Fig. 5 Ephemeris runs corresponding to 40 cycles periodic orbits in the simplified problem.

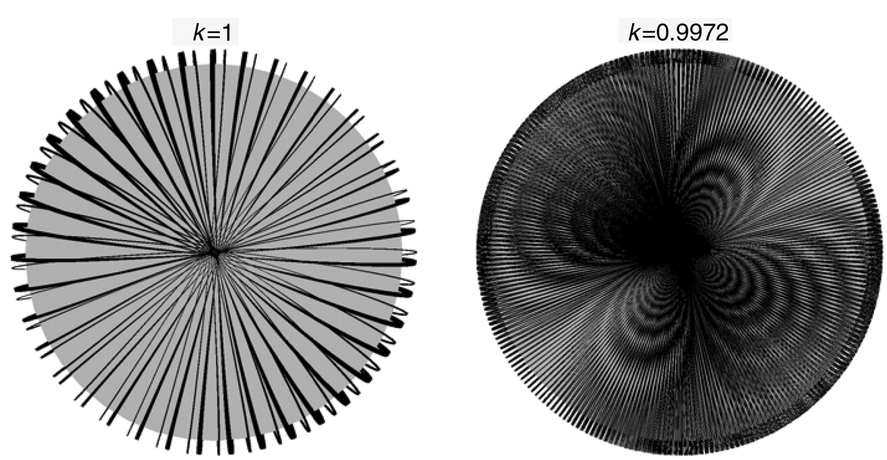

Fig. 6 Left: 128 days lifetime orbit with repeat ground-track. Right: 163 days lifetime.

Conclusions

The computation of periodic solutions of a Hill perturbed problem reveals as a versatile and fast procedure for searching for long lifetime repeat ground-track orbits around Europa. Close to Europa, the repetition cycle is long enough to satisfy the requirements of a science orbit. Notably, we have found science orbits lasting more than six months in the ephemeris model. Further, our solutions are quite robust, and the lifetimes average to about four months in a wide region around the optimum set of initial conditions. These lifetimes are consistent with previous ephemeris studies.

The one-dimensional scaling we used in the ephemeris model might also be useful answering two questions that have not been addressed in previous analytic studies. Namely, given analytically derived initial conditions in a simpler dynamical model, how long can we expect the same orbit to last in a true ephemeris? And how isolated is the longest lifetime orbit, or how large is the radius of the long-lifetime phase space?

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